CLASS X: MATHS
Chapter 5: Arithmetic Progressions

Questions and Solutions | Exercise 5.1 - NCERT Books

Q1. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

(i) The taxi fare after each km when the fare is Rs. 15 for the first km and Rs. 8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes 1/4 of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs Rs. 150 for the first metre and rises by Rs. 50 for each subsequent metre.

(iv) The amount of money in the account every year, when Rs. 10000 is deposited at compound interest at 8% per annum.

Sol. (i) t_n denotes the taxi fare (in Rs.) for the first n km.

Now, $t_1 = 15$,

$t_2 = 15 + 8 = 23$,

$t_3 = 23 + 8 = 31$,

$t_4 = 31 + 8 = 39$,....

List of fares after 1 km, 2 km, 3 km, 4 km, ... respectively is 15, 23, 31, 39, ... (in Rs.).

Here, $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = ... = 8$.

Thus, the list forms an AP.

(ii) $t_1 = x$ units ; $t_2 = x - \frac{1}{4}x = \frac{3}{4}x$ units ;

$$t_3 = \frac{3}{4}x - \frac{1}{4}\left(\frac{3}{4}x\right) = \frac{3}{4}x - \frac{3}{16}x = \frac{9}{16}x \text{ units}$$

$$t_4 = \frac{9}{16}x - \frac{1}{4}\left(\frac{9}{16}x\right) = \frac{27}{64}x \text{ units}$$

The list of numbers is x, $\frac{3}{4}x, \frac{9}{16}x, \frac{27}{64}x, ...$

It is not an AP because $t_2 - t_1 \neq t_3 - t_2$.

(iii) Cost of digging for first metre = 150

Cost of digging for first 2 metres

= 150 + 50 = 200

Cost of digging for first 3 metres

= 200 + 50 = 250

Cost of digging for first 4 metres

= 250 + 50 = 300

Clearly, 150, 200, 250, 300.... forms an A.P.

Here, $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = ... = 50$.

Thus, the list forms an AP.

(iv) We know that if Rs P is deposited at r% compound interest per annum for n years, our money will be $P\left(1+\frac{r}{100}\right)^n$ after n years.

Therefore, after every year, our money will be

$$10000\left(1+\frac{8}{100}\right),\ 10000\left(1+\frac{8}{100}\right)^2,$$

$$10000\left(1+\frac{8}{100}\right)^3,\ 10000\left(1+\frac{8}{100}\right)^4, ...$$

Clearly, adjacent terms of this series do not have the same difference between them. Therefore, this is not an A.P.

Q2. Write first four terms of the AP, when the first term a and the common difference d are given as follows:

(i) a = 10, d = 10 (ii) a = – 2 d = 0

(iii) a = 4, d = – 3 (iv) a = – 1, d = 1/2

(v) a = – 1.25, d = – 0.25

Sol. (i) $t_1 = a = 10,$

$t_2 = 10 + d = 10 + 10 = 20,$

$t_3 = 20 + d = 20 + 10 = 30,$

$t_4 = 30 + d = 30 + 10 = 40,....$

Thus, the AP is 10, 20, 30, 40, ...

(ii) Given $a = -2$ and $d = 0$

$t_1 = -2,\ t_2 = -2 + 0 = -2,$

$t_3 = -2 + 0 = -2,\ t_4 = -2 + 0 = -2,\ ....$

Thus, the AP is –2, – 2, – 2, – 2, ...

(iii) $a = 4,\ d = -3$

$t_1 = a = 4$

$t_2 = a_1 + d = 4 - 3 = 1$

$t_3 = a_2 + d = 1 - 3 = -2$

$t_4 = a_3 + d = -2 - 3 = -5$

Therefore, the series will be 4, 1, –2 –5 …

First four terms of this A.P. will be 4, 1, –2 and –5.

(iv) $a = -1,\ d = \frac{1}{2}$

$t_1 = a = -1$

$t_2 = a_1 + d = -1 + \frac{1}{2} = -\frac{1}{2}$

$t_3 = a_2 + d = -\frac{1}{2} + \frac{1}{2} = 0$

$t_4 = a_3 + d = 0 + \frac{1}{2} = \frac{1}{2}$

Clearly, the series will be

$-1, -\frac{1}{2}, 0, \frac{1}{2}$ … … … … .

First four terms of this A.P. will be

$-1, -\frac{1}{2}$, 0 and $-\frac{1}{2}$

(v) $a = -1.25, d = -0.25$

$t_1 = a = -1.25$

$t_2 = a_1 + d = -1.25 - 0.25 = -1.50$

$t_3 = a_2 + d = -1.50 - 0.25 = -1.75$

$t_4 = a_3 + d = -1.75 - 0.25 = -2.00$

Clearly, the series will be –1.25, –1.50, –1.75, –2.00 … … ....

First four terms of this A.P. will be –1.25, –1.50, –1.75 and –2.00.

Q3. For the following APs, write the first term and the common difference :

(i) 3, 1, – 1, –3, ...

(ii) –5, – 1, 3, 7, ...

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, ...$

(iv) 0.6, 1.7, 2.8, 3.9, ...

Sol. (i) $a = 3, d = t_2 - t_1 = 1 - 3 = -2,$

i.e., $d = -2$

(ii) $a = -5, d = 4$

(iii) $a = \frac{1}{3}$

$d = t_2 - t_1 = \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

(iv) 0.6, 1.7, 2.8, 3.9 …

$a = 0.6$

$d = t_2 - t_1$

$= 1.7 - 0.6$

$= 1.1$

Q4. Which of the following are APs ? If they form an AP, find the common difference d and write three more terms.

(i) 2, 4, 8, 16,....

(ii) $2, \frac{5}{2}, 3, \frac{7}{2}, \ldots.$

(iii) – 1.2, – 3.2, – 5.2, – 7.2, ...

(iv) – 10, – 6, – 2, 2, ...

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$

(vi) 0.2, 0.22, 0.222, 0.2222, ...

(vii) 0, – 4, – 8, – 12, ...

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \ldots.$

(ix) 1, 3, 9, 27, ...

(x) a, 2a, 3a, 4a, ...

(xi) a, a^2, a^3, a^4, ...

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots.$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots.$

(xiv) $1^2, 3^2, 5^2, 7^2, \ldots$

(xv) $1^2, 5^2, 7^2, 73, \ldots.$

Sol. (i) Not an AP because $t_2 - t_1 = 2$

and $t_3 - t_2 = 8 - 4 = 4$,

i.e., $t_2 - t_1 \neq t_3 - t_2$.

(ii) It is an AP. $a = 2, d = 1/2$

$[\because t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = 1/2]$

$t_5 = \frac{7}{2} + \frac{1}{2} = 4, t_6 = 4 + \frac{1}{2} = \frac{9}{2},$

$t_7 = \frac{9}{2} + \frac{1}{2} = 5.$

(iii) We have : –1.2, –3.2, –5.2, –7.2, ...

$\therefore t_1 = -1.2, t_2 = -3.2, t_3 = -5.2, t_4 = -7.2$

$t_2 - t_1 = -3.2 + 1.2 = -2$

$t_3 - t_2 = -5.2 + 3.2 = -2$

$t_4 - t_3 = -7.2 + 5.2 = -2$

$\because t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = -2$

$\Rightarrow d = -2$

$\therefore$ The given numbers from an A.P. such that $d = -2$.

Now, $t_5 = t_4 + (-2) = -7.2 + (-2) = -9.2,$

$t_6 = t_5 + (-2) = -9.2 + (-2) = -11.2$

and $t_7 = t_6 + (-2)$

$= -11.2 + (-2) = -13.2$

Thus, $d = -2$ and $t_5 = -9.2, t_6 = -11.2$ and $t_7 = -13.2$

(iv) It is an AP.

$a = -10, d = 4, t_5 = 6, t_6 = 10, t_7 = 14.$

(v) It is an AP.

$a = 3, d = \sqrt{2}$

$t_5 = 3 + 3\sqrt{2} + \sqrt{2} = 3 + 4\sqrt{2},$

$t_6 = 3 + 5\sqrt{2}, t_7 = 3 + 6\sqrt{2}.$

(vi) It is not AP.

$t_2 - t_1 = 0.22 - 0.2 = 0.02,$

$t_3 - t_2 = 0.222 - 0.22 = 0.002, \ldots$

i.e., $t_2 - t_1 \neq t_3 - t_2.$

(vii) We have : 0, –4, –8, –12, ...

$\therefore\ t_1 = 0, t_2 = -4, t_3 = -8, t_4 = -12$

$t_2 - t_1 = -4 - 0 = -4$

$t_3 - t_2 = -8 + 4 = -4$

$t_4 - t_3 = -12 + 8 = -4$

$\because\ t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = -4 \Rightarrow d = -4$

$\therefore$ The given numbers from an A.P.

Now, $t_5 = t_4 + (-4) = -12 + (-4) = -16$

$t_6 = t_5 + (-4) = -16 + (-4) = -20$

$t_7 = t_6 + (-4) = -20 + (-4) = -24$

Thus, $d = -4$ and $t_5 = -16, t_6 = -20, \quad t_7 = -24.$

(viii) We have : $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, ...$

$\therefore\ t_1 = t_2 = t_3 = t_4 = -\frac{1}{2}$

$t_2 - t_1 = 0, t_3 - t_2 = 0, t_4 - t_3 = 0 \Rightarrow d = 0$

$\therefore$ The given numbers from an A.P.

Now, $t_5 = -\frac{1}{2} + 0 = -\frac{1}{2}$

$t_6 = -\frac{1}{2} + 0 = -\frac{1}{2}$, $t_7 = -\frac{1}{2} + 0 = -\frac{1}{2}$

Thus, $d = 0$ and $t_5 = -\frac{1}{2}$, $t_6 = -\frac{1}{2}$, $t_7 = -\frac{1}{2}$

(ix) Not an A.P. Here, $t_2 - t_1 \neq t_3 - t_2$.

(x) We have : a, 2a, 3a, 4a, ...

$\therefore$ $t_1 = a$, $t_2 = 2a$, $t_3 = 3a$, $t_4 = 4a$

$t_2 - t_1 = 2a - a = a$,

$t_3 - t_2 = 3a - 2a = a$ and

$t_4 - t_3 = 4a - 3a = a$

$\because$ $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = a$

$\Rightarrow$ $d = a$

$\therefore$ The given numbers from an A.P.

Now, $t_5 = t_4 + a = 4a + a = 5a$, $t_6 = t_5 + a$

$= 5a + a = 6a$ and $t_7 = t_6 + a = 6a + a = 7a$

Thus, $d = a$ and $t_5 = 5a$, $t_6 = 6a$, $t_7 = 7a$

(xi) Not an AP if $a \neq 1$.

Here, $t_2 - t_1 = a^2 - a = a(1 - a)$,

$t_3 - t_2 = a^3 - a^2 = a^2(1 - a)$

$t_3 - t_2 \neq t_2 - t_1$ when $a \neq 1$.

It will be an AP if $a = 1$.

Hence, the given sequence is an AP only when $a = 1$.

In this case, first term = 1,

common difference = 0

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots$ can be rewritten as

$\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, \ldots\ldots$

$a = \sqrt{2}, d = \sqrt{2}$

$t_5 = 5\sqrt{2},\ t_6 = 6\sqrt{2},\ t_7 = 7\sqrt{2},$

i.e., $t_5 = \sqrt{50}$, $t_6 = \sqrt{72},\ t_7 = \sqrt{98}.$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots$ can be rewritten as

$\sqrt{3},\ \sqrt{2} \times \sqrt{3},\ 3,\ 2\sqrt{3}, \ldots$

Here, $t_2 - t_1 \neq t_3 - t_2$

Therefore, the given list is not an AP.

(xiv) We have $1^2, 3^2, 5^2, 7^2, \ldots\ldots\ldots$

$$\therefore \left.\begin{aligned} t_1 &= 1^2 = 1 \\ t_2 &= 3^2 = 9 \end{aligned}\right\} \Rightarrow t_2 - t_1 = 9 - 1 = 8$$

$$\left.\begin{aligned} t_3 &= 5^2 = 25 \\ t_4 &= 7^2 = 49 \end{aligned}\right\} \Rightarrow t_4 - t_3 = 49 - 25 = 24$$

$\because t_2 - t_1 \neq t_4 - t_3$

$\therefore$ The given numbers do not form an A.P.

(xv) $1^2, 5^2, 7^2, 73, \ldots$ can be rewritten as $1, 25, 49, 73, \ldots.$

Here, $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = \ldots. = 24$

Hence, it is an AP.

$\therefore\ t_5 = 97,\quad t_6 = 121,\ \ t_7 = 145$

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Q1. Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n, the n^{th} term of the AP.

	a	d	n	a_n
(i)	7	3	8	...
(ii)	–18	...	10	0
(iii)	...	–3	18	–5
(iv)	–18.9	2.5	...	3.6
(v)	3.5	0	105	...

Sol. (i) $a = 7, d = 3, n = 8$

$a_8 = a + 7d = 7 + 7 \times 3 = 28.$

Hence, $a_8 = 28.$

(ii) $a = –18, n = 10, a_n = 0, d = ?$

$a_n = a + (n – 1) d$

$0 = – 18 + (10 – 1) d$

$18 = 9d \quad \Rightarrow \quad d = \frac{18}{9} = 2$

Hence, $d = 2$

(iii) $d = –3, n = 18, a_n = –5$

$a_n = a + (n – 1) d$

$–5 = a + (18 – 1) (–3)$

$–5 = a + (17) (–3)$

$–5 = a – 51$

$a = 51 – 5 = 46$

Hence, $a = 46$

(iv) $a = – 18.9, d = 2.5$

$t_n = 3.6$

$\Rightarrow \quad a + (n – 1) d = 3.6$

$\Rightarrow \quad – 18.9 + (n – 1) \times (2.5) = 3.6$

$\Rightarrow \quad (n – 1) \times (2.5) = 3.6 + 18.9 = 22.5$

$\Rightarrow$ $n - 1 = \frac{22.5}{2.5} = \frac{225}{25} = 9$

$\Rightarrow$ $n = 10$

(v) $a = 3.5$, $d = 0$, $n = 105$

Then $a_{105} = a + 104\,d = 3.5 + 0 = 3.5$

Q2. Choose the correct choice in the following and justify

(i) 30th term of the AP : 10, 7, 4,.... is

(A) 97 (B) 77 (C) – 77 (D) – 87

(ii) 11th term of the AP : $-3, -\frac{1}{2}, 2, \ldots$ is

(A) 28 (B) 22 (C) – 38 (D) $-48\frac{1}{2}$

Sol. (i) $a = 10$, $d = -3$

$t_{30} = a + 29d = 10 + 29 \times (-3)$

$= 10 - 87 = -77$

Hence, the correct option is (C)

(ii) $a = -3$, $d = 5/2$

$t_{11} = a + 10d = -3 + 10 \times 5/2 = 22$

Hence, the correct option is (B)

Q3. In the following APs, find the missing terms in the boxes :

(i) 2, $\square$, 26

(ii) $\square$, 13, $\square$, 3

(iii) 5, $\square$, $\square$, $9\frac{1}{2}$

(iv) – 4, $\square$, $\square$, $\square$, $\square$, 6

(v) $\square$, 38, $\square$, $\square$, $\square$, – 22

Sol. (i) $a = 2$, $a + 2\,d = 26$ $\Rightarrow 2 + 2d = 26$

$\Rightarrow \; 2d = 26 - 2 = 24 \qquad \Rightarrow d = 12$

Then the missing term

$t_2 = a + d = 2 + 12 = 14$

(ii) $a + d = 13 \qquad ...(1)$

$a + 3\,d = 3 \qquad ...(2)$

Subtracting (1) from (2), we get

$(a + 3d) - (a + d) = 3 - 13$

$\Rightarrow \; 2d = -10 \quad \Rightarrow \; d = -5$

from (1), $a - 5 = 13$

$\Rightarrow \; a = 18$

Therefore, the first missing term is 18

The next missing term

$t_3 = t_2 + d = 13 + (-5) = 8$

(iii) $a = 5$

$a_4 = 9\frac{1}{2} = \frac{19}{2} \qquad a + 3d = \frac{19}{2}$

$\frac{19}{2} = 5 + 3d$

$\frac{19}{2} - 5 = 3d$

$\frac{9}{2} = 3d \qquad d = \frac{3}{2}$

$a_2 = a + d = 5 + \frac{3}{2} = \frac{13}{2}$

$a_3 = a + 2d = 5 + 2\left(\frac{3}{2}\right) = 8$

Therefore, the missing terms are $\frac{13}{2}$ and 8 respectively.

(iv) $a = -4$

$a_6 = 6$

$a + 5d = 6$

$6 = -4 + 5d$

$10 = 5d$
$d = 2$
$a_2 = a + d = -4 + 2 = -2$
$a_3 = a + 2d = -4 + 2(2) = 0$
$a_4 = a + 3d = -4 + 3(2) = 2$
$a_5 = a + 4d = -4 + 4(2) = 4$
Therefore, the missing terms are –2, 0, 2, and 4 respectively.

(v) $a_2 = 38$
$a_6 = -22$
$38 = a + d$ ....(1)
$-22 = a + 5d$ ....(2)
On subtracting equation (1) from (2), we obtain
$-22 - 38 = 4d$
$-60 = 4d$
$d = -15$
$a = a_2 - d = 38 - (-15) = 53$
$a_3 = a + 2d = 53 + 2(-15) = 23$
$a_4 = a + 3d = 53 + 3(-15) = 8$
$a_5 = a + 4d = 53 + 4(-15) = -7$
Therefore, the missing terms are 53, 23, 8, and –7 respectively.

Q4. Which term of the AP : 3, 8, 13, 18, ... is 78?

Sol. $a = 3, d = 5$
Let $t_n = 78$
$\Rightarrow a + (n - 1)d = 78$
$\Rightarrow 3 + (n - 1) \times 5 = 78 \Rightarrow 5n - 2 = 78$
$\Rightarrow 5n = 80 \qquad \Rightarrow n = 16$
Hence, $t_{16} = 78$

Q5. Find the number of terms in each of the following AP's :
(i) 7, 13, 19. ....., 205
(ii) $18, 15\frac{1}{2}, 13, \ldots, -47$

Sol. (i) $a = 7, d = 6,$

$t_n = 205$

$\Rightarrow a + (n - 1) d = 205$

$\Rightarrow 7 + (n - 1) \times 6 = 205 \quad \Rightarrow 6n + 1 = 205$

$\Rightarrow 6n = 204 \quad \Rightarrow n = 34$

Hence, 34 terms

(ii) $a = 18$

$d = a_2 - a_1 = 15\frac{1}{2} - 18$

$d = \frac{31-36}{2} = -\frac{5}{2}$

Let there are n terms in this A.P.

Therefore, $a_n = -47$ and we know that

$a_n = a + (n - 1)d$

$-47 = 18 + (n-1)\left(-\frac{5}{2}\right)$

$-47 = 18 + (n-1)\left(-\frac{5}{2}\right)$

$-65 = (n-1)\left(-\frac{5}{2}\right)$

$(n-1) = \frac{-130}{-5}$

$(n - 1) = 26$

$n = 27$

Therefore, this given A.P. has 27 terms in it.

Q6. Check whether – 150 is a term of the AP : 11, 8, 5, 2, ...

Sol. $a = 11, d = -3$

Let if possible $t_n = -150$

$\Rightarrow a + (n - 1) d = -150$

$\Rightarrow 11 + (n - 1) \times (-3) = -150$

$\Rightarrow 11 - 3n + 3 = -150$

$\Rightarrow 14 - 3n = -150$

$\Rightarrow$ $3n = 14 + 150 = 164$

$\Rightarrow$ $n = \frac{164}{3} = 54\frac{2}{3}$

It is not possible because n is to be natural number.

Hence, – 150 cannot be a term of the AP.

Q7. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

Sol. Given that,

$a_{11} = 38$

$a_{16} = 73$

We know that,

$a_n = a + (n - 1)\,d$

$a_{11} = a + (11 - 1)\,d$

$38 = a + 10d$ ......(1)

Similarly,

$a_{16} = a + (16 - 1)\,d$

$73 = a + 15d$ ......(2)

On subtracting (1) from (2), we obtain

$35 = 5d$

$d = 7$

From equation (1),

$38 = a + 10 \times (7)$

$38 - 70 = a$

$a = -32$

$a_{31} = a + (31 - 1)\,d$

$= -32 + 30\,(7)$

$= -32 + 210$

$= 178$

Hence, 31^{st} term is 178.

Q8. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

Sol. $t_3 = 12$, t_{50} (last term) = 106

$\Rightarrow$ $a + 2d = 12$ ...(1)

and $a + 49d = 106$ ...(2)

Subtracting (1) from (2), we get

$47d = 106 - 12 = 94 \Rightarrow d = 2$

From (1), $a + 2 \times 2 = 12$ $\Rightarrow a = 8$

$t_{29} = a + 28d = 8 + 28 \times 2 = 64$

Q9. If the 3rd and 9th terms of an AP are 4 and – 8 respectively, which term of this AP is zero?

Sol. Given that,

$a_3 = 4$

$a_9 = -8$

We know that,

$a_n = a + (n - 1)\,d$

$a_3 = a + (3 - 1)\,d$

$4 = a + 2d$ ......(I)

$a_9 = a + (9 - 1)\,d$

$-8 = a + 8d$ ......(II)

On subtracting equation (I) from (II), we obtain

$-12 = 6d$

$d = -2$

From equation (I), we obtain

$4 = a + 2\,(-2)$

$4 = a - 4$

$a = 8$

Let n^{th} term of this A.P. be zero.

$a_n = a + (n - 1) d$

$0 = 8 + (n - 1) (-2)$

$0 = 8 - 2n + 2$

$2n = 10$

$n = 5$

Hence, 5^{th} term of this A.P. is 0.

Q10. The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

Sol. $a_{17} - a_{10} = 7$

$(a + 16d) - (a + 9d) = 7$

$7d = 7$

$d = 1$

Therefore, the common difference is 1.

Q11. Which term of the AP : 3, 15, 27, 39, ... will be 132 more than its 54th term?

Sol. $a = 3, d = 12$

Let us suppose $t_n = t_{54} + 132$

$\Rightarrow$ $a + (n - 1) d = a + 53 d + 132$

$\Rightarrow$ $(n - 1) d - 53 d = 132$

$\Rightarrow$ $\{n - 1 - 53) d = 132$

$\Rightarrow$ $(n - 54) \times 12 = 132$

$\Rightarrow$ $n - 54 = 11$

$\Rightarrow$ $n = 65$

Hence, t_{65} is 132 more than t_{54}.

Q12. Two APs have the same common difference. The difference between their 100 th terms is 100, what is the difference between their 1000th terms?

Sol. Let the two APs with same common difference d be

$a, a + d, a + 2d, \ldots.$

$b, b + d, b + 2d, \ldots. \ (a > b)$

We are given that

$\{100\text{th term of the first AP}\}$

$- \{100\text{th term of the second AP}\} = 100$

$\Rightarrow \{a + 99d\} - \{b + 99d\} = 100$

$\Rightarrow a - b = 100 \quad \ldots(1)$

Now, {1000th term of the first AP)

$- \{1000\text{th term of the second AP}\}$

$= \{a + 999d\} - \{b + 999d\} = a - b = 100$

{By (1)}

Q13. How many three-digit numbers are divisible by 7?

Sol. First three-digit number that is divisible by 7 = 105

Next number = 105 + 7 = 112

Therefore, 105, 112, 119, …

All are three digit numbers which are divisible by 7 and thus, all these are terms of an A.P. having first term as 105 and common difference as 7.

The maximum possible three-digit number is 999. When we divide it by 7, the remainder will be 5. Clearly, 999 - 5 = 994 is the maximum possible three-digit number that is divisible by 7.

The series is as follows.

105, 112, 119, ..., 994

Let 994 be the n^{th} term of this A.P.

$a = 105$

$d = 7$

$a_n = 994$

$n = ?$

$a_n = a + (n - 1)\, d$

$994 = 105 + (n - 1)\, 7$

$889 = (n - 1)\, 7$

$(n - 1) = 127$

$n = 128$

Therefore, 128 three-digit numbers are divisible by 7.

Q14. How many multiples of 4 lie between 10 and 250?

Sol. The multiples of 4 between 10 and 250 are 12, 16, 20, 24...., 248.

Let these numbers be n.

$a = 12, d = 4$

$t_n = 248$

$\Rightarrow\ a + (n - 1)\, d = 248$

$\Rightarrow\ 12 + (n - 1) \times 4 = 248$

$\Rightarrow\ 4n + 8 = 248 \Rightarrow n = 60.$

Q15. For what value of n, are the nth terms of two APs 63, 65, 67, ... and 3, 10, 17, ... equal?

Sol. Two APs are 63, 65, 67, ..., 3, 10, 17, ...

From (1), First term = 63 and common difference = 2.

Its nth term $= 63 + (n - 1) \times 2 = 2n + 61$.

From (2), First term = 3 and common difference = 7

Its nth term $= 3 + (n - 1) \times 7 = 7n - 4$

Putting $7n - 4 = 2n + 61$

$\Rightarrow\ 7n - 2n = 61 + 4 \Rightarrow 5n = 65 \Rightarrow n = 13$

Q16. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

Sol. $a_3 = 16$

$a + (3 - 1)\, d = 16$
$a + 2d = 16$ .... (1)
$a_7 - a_5 = 12$
$[a + (7 - 1)\, d] - [a + (5 - 1)\, d] = 12$
$(a + 6d) - (a + 4d) = 12$
$2d = 12$
$d = 6$
From equation (1), we obtain
$a + 2\,(6) = 16$
$a + 12 = 16$
$a = 4$
Therefore, A.P. will be
4, 10, 16, 22, …

Q17. Find the 20th term from the last term of the AP 3, 8, 13, ..., 253.

Sol. The AP is 3, 8, 13, ..., 253
Its first term = 3 and the common difference = 5.
Now, the AP in the reverse order will have the first term = 253 and the common difference = – 5.
The 20th term from the end of the AP (1)
= The 20 term of the AP in the reverse order
$= a + 19d$
$= 253 + 19 \times (-5) = 253 - 95 = 158.$

Q18. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

Sol. $t_4 + t_8 = 24$; $t_6 + t_{10} = 44$
$\Rightarrow$ $(a + 3d) + (a + 7d) = 24$;
$(a + 5d) + (a + 9d) = 44$
$\Rightarrow$ $2a + 10d = 24$; $2a + 14d = 44$

We have $a + 5d = 12$ ...(1)

and $a + 7d = 22$ ...(2)

Subtracting (1) from (2), we get

$2d = 10 \Rightarrow d = 5$

From (i) $a + 5 \times 5 = 12$, $a = -13$

$t_1 = -13$, $t_2 = -8$, $t_3 = -3$

Q19. Subba Rao started work in 1995 at an annual salary of Rs. 5000 and received an increment of Rs. 200 each year. In which year did his income reach Rs. 7000 ?

Sol. It can be observed that the incomes that Subba Rao obtained in various years are in A.P. as every year, his salary is increased by Rs. 200.

Therefore, the salaries of each year after 1995 are

5000, 5200, 5400, ...

Here, $a = 5000$

$d = 200$

Let after n^{th} year, his salary be Rs 7000.

Therefore, $a_n = a + (n - 1) d$

$7000 = 5000 + (n - 1) 200$

$200(n - 1) = 2000$

$(n - 1) = 10$

$n = 11$

Therefore, in 11th year, his salary will be Rs 7000.

Q20. Ramkali saved Rs. 5 in the first week of a year and then increased her weekly savings by Rs. 1.75. If in the nth week, her weekly savings become Rs. 20.75, find n.

Sol. t_1 = Rs. 5 (savings in the Ist week)

t_2 = Rs. 5 + Rs. 1.75 = Rs. 6.75

(savings in the 2nd week)

t_3 = Rs. 6.75 + Rs. 1.75 = Rs. 8.50

(savings in the 3rd week)

....

t_n = Rs. 20.75

$\Rightarrow$ a + (n – 1) d = 20.75

$\Rightarrow$ 5 + (n – 1) × 1.75 = 20.75

$\Rightarrow$ (n – 1) × 1.75 = 15.75

$\Rightarrow n - 1 = \frac{15.75}{1.75} = \frac{1575}{175} = 9 \Rightarrow n = 10$

Hence, in the 10th week, Ramkali's savings will be Rs. 20.75

Questions and Solutions | Exercise 5.3 - NCERT Books

Q1. Find the sum of the following APs :

(i) 2, 7, 12, ... to 10 terms.

(ii) – 37, – 33, – 29, ... to 12 terms.

(iii) 0.6, 1.7, 2.8, ... to 100 terms

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots$ to 11 terms

Sol. (i) a = 2, d = 5

$S_{10} = \frac{10}{2}\ \{2a + 9d\}$

$\left(\because S_n = \frac{n}{2}\{2a + (n-1)d\}\right)$

= 5 × {2 × 2 + 9 × 5) = 5 × 49 = 245

(ii) a = – 37, d = 4

$S_{12} = \frac{12}{2}\ \{2a + 11d\}$

$= 6 \times \{2(-37) + 11 \times 4\}$

$= 6 \times \{-74 + 44\} = -180$

(iii) 0.6, 1.7, 2.8,....., to 100 terms

For this A.P.,

$a = 0.6$

$d = a_2 - a_1 = 1.7 - 0.6 = 1.1$

$n = 100$

$$S_{100} = \frac{100}{2}[2(0.6) + (100-1)1.1]$$

$= 50[1.2 + (99) \times (1.1)]$

$= 50[110.1]$

$= 110.1$

$= 5505$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$, ... , to 11 terms

For this A.P.,

$a = \frac{1}{15}$

$n = 11$

$d = a_2 - a_1 = \frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}$

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{11} = \frac{11}{2}\left[2\left(\frac{1}{15}\right) + (11-1)\frac{1}{60}\right]$$

$$= \frac{11}{2}\left[\frac{2}{15} + \frac{10}{60}\right]$$

$$= \frac{11}{2}\left[\frac{2}{15}+\frac{1}{6}\right] = \frac{11}{2}\left[\frac{4+5}{30}\right]$$

$$= \left(\frac{11}{2}\right)\left(\frac{9}{30}\right) = \frac{33}{20}$$

Q2. Find the sums given below :

(i) $7 + 10\frac{1}{2} + 14 + \ldots + 84$.

(ii) $34 + 32 + 30 + \ldots + 10$

(iii) $-5 + (-8) + (-11) + \ldots + (-230)$.

Sol. (i) $a = 7, d = 10\frac{1}{2} - 7 = 3\frac{1}{2} = \frac{7}{2}$

$\ell = t_n = 84 \Rightarrow a + (n-1)d = 84$

$\Rightarrow 7 + (n-1) \times \frac{7}{2} = 84$

$\Rightarrow (n-1) \times \frac{7}{2} = 77$

$\Rightarrow n - 1 = 77 \times \frac{2}{7} = 22$

$\Rightarrow n = 23$

The sum $= \frac{n}{2}\{a + t_n) = \frac{23}{2}\{7 + 84)$

$= \frac{23}{2} \times 91 = \frac{2093}{2} = 1046\frac{1}{2}$

(ii) $34 + 32 + 30 + \ldots\ldots. + 10$

$a = 34$

$d = a_2 - a_1 = 32 - 34 = -2$

$\ell = 10$

Let 10 be the nth term of this A.P.

$\ell = a + (n-1)d$

$10 = 34 + (n-1)(-2)$

$-24 = (n-1)(-2)$

$12 = n - 1$

$n = 13$

$S_n = \frac{n}{2}(a + \ell)$

$= \frac{13}{2}(34 + 10)$

$= \frac{13 \times 44}{2} = 13 \times 22 = 286$

(iii) $(-5) + (-8) + (-11) + \ldots\ldots\ldots + (-230)$

For this A.P.,

$a = -5$

$\ell = -230$

$d = a_2 - a_1 = (-8) - (-5)$

$= -8 + 5 = -3$

Let -230 be the n^{th} term of this A.P.

$\ell = a + (n - 1)d$

$-230 = -5 + (n - 1)(-3)$

$-225 = (n - 1)(-3)$

$(n - 1) = 75$

$n = 76$

And, $S_n = \frac{n}{2}(a + \ell)$

$= \frac{76}{2}[(-5) + (-230)]$

$= 38(-235)$

$= -8930$

Q3. In an AP :

(i) Given $a = 5$, $d = 3$, $a_n = 50$, find n and S_n.

(ii) Given $a = 7$, $a_{13} = 35$, find d and S_{13}.

(iii) Given $a_{12} = 37$, $d = 3$, find a and S_{12}.

(iv) Given $a_3 = 15$, $S_{10} = 125$, find d and a_{10}.

(v) Given $d = 5$, $S_9 = 75$, find a and a_9.

(vi) Given $a = 2$, $d = 8$, $S_n = 90$, find n and a_n.

(vii) Given $a = 8$, $a_n = 62$, $S_n = 210$, find n and d.

(viii) Given $a_n = 4$, $d = 2$, $S_n = -14$, find n and a.

(ix) Given $a = 3$, $n = 8$, $S = 192$, find d.

(x) Given $\ell = 28$, $S = 144$, and there are total 9 terms. Find a.

Sol. (i) $a = 5$, $d = 3$, $a_n = 50$

$\Rightarrow$ $a + (n - 1)\, d = 50$

$\Rightarrow$ $5 + (n - 1)(3) = 50$

$\Rightarrow$ $5 + 3n - 3 = 50$ or $3n = 48$ or $n = 16$

$$S_{16} = \frac{16}{2}\{2a + 15d\}$$

$= 8\{10 + 15 \times 3\} = 440$

(ii) $a = 7$, $a_{13} = 35$

$\therefore\ a_{13} = a + (13 - 1)\, d$

$35 = 7 + 12\, d$

$35 - 7 = 12d$

$28 = 12d$

$$d = \frac{7}{3}$$

$$S_{13} = \frac{n}{2}[a + a_{13}]$$

$= \frac{13}{2}[7 + 35]$

$= \frac{13 \times 42}{2} = 13 \times 21$

$= 273$

(iii) $a_{12} = 37$, $d = 3$

$a_{12} = a + (12 - 1)3$

$37 = a + 33$

$a = 4$

$S_n = \frac{n}{2}[a + a_n]$

$S_{12} = \frac{12}{2}[4 + 37]$

$S_{12} = 6(41)$

$S_{12} = 246$

(iv) $a_3 = 15$, $S_{10} = 125$

$a_3 = a + (3 - 1)d$

$15 = a + 2d$ .....(i)

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{10} = \frac{10}{2}[2a + (n - 1)d]$

$125 = 5(2a + 9d)$

$25 = 2a + 9d$ ....(ii)

On multiplying equation (i) by 2, we obtain

$30 = 2a + 4d$ .....(iii)

On subtracting equation (iii) from (ii), we obtain

$-5 = 5d$

$d = -1$

From equation (i),

$15 = a + 2(-1)$

$15 = a - 2$

$a = 17$

$a_{10} = a + (10 - 1)d$

$a_{10} = 17 + (9)(-1)$

$a_{10} = 17 - 9 = 8$

(v) $d = 5, S_9 = 75$

$S_9 = \frac{9}{2}[2a + (9 - 1)5]$

$75 = \frac{9}{2}(2a + 40)$

$25 = 3(a + 20)$

$25 = 3a + 60$

$3a = 25 - 60$

$a = \frac{-35}{3}$

$a_9 = a + (9 - 1)(5)$

$= \frac{-35}{3} + 8(5)$

$= \frac{-35}{3} + 40$

$= \frac{-35 + 120}{3} = \frac{85}{3}$

(vi) $a = 2, d = 8, S_n = 90$

$\Rightarrow \frac{n}{2}\{2a + (n-1)d\} = 90$

$\Rightarrow \frac{n}{2}\{4 + (n-1) \times 8\} = 90$

$\Rightarrow \frac{n}{2} \times \{8n - 4\} = 90$

$\Rightarrow 4n^2 - 2n - 90 = 0$

$\Rightarrow 2n^2 - n - 45 = 0$

$\Rightarrow 2n^2 - 10n + 9n - 45 = 0$

$\Rightarrow 2n(n-5) + 9(n-5) = 0$

$\Rightarrow (n-5)(2n+9) = 0$

$\Rightarrow n - 5 = 0 \qquad (\because 2n + 9 \neq 0)$

$\Rightarrow n = 5$

$a_n = a_5 = a + 4d = 2 + 4 \times 8 = 34$

$\Rightarrow a_n = 34$

(vii) $a = 8, a_n = 62, S_n = 210$

$210 = \frac{n}{2}[8 + 62]$

$210 = \frac{n}{2}(70)$

$n = 6$

$a_n = a + (n-1)d$

$62 = 8 + (6-1)d$

$62 - 8 = 5d$

$54 = 5d$

$d = \frac{54}{5}$

(viii) $a_n = 4, d = 2, S_n = -14$

Now, $a_n = 4 \Rightarrow a + (n-1)d = 4$

$\Rightarrow a + (n-1)(2) = 4$

$\Rightarrow a = 6 - 2n \qquad ...(1)$

$S_n = -14$

$\Rightarrow \frac{n}{2}\{2a + (n-1)d\} = -14$

$\Rightarrow \frac{n}{2}\{2(6-2n) + (n-1)(2)\} = -14 \quad \{By(1)\}$

$\Rightarrow \frac{n}{2}\{12 - 4n + 2n - 2) = -14$

$\Rightarrow \frac{n}{2}\{10 - 2n\} = -14$

$\Rightarrow n(n-5) = 14$

$\Rightarrow n^2 - 5n - 14 = 0$

$\Rightarrow n^2 - 7n + 2n - 14 = 0$

$\Rightarrow n(n-7) + 2(n-7) = 0$

$\Rightarrow (n-7)(n+2) = 0$

$\Rightarrow n = 7$

From (1), $a = 6 - 2 \times 7 = -8$

$a = -8$

(ix) $a = 3, n = 8, S = 192$

$192 = \frac{8}{2}[2 \times 3 + (8-1)d]$

$192 = 4[6 + 7d]$

$48 = 6 + 7d$

$42 = 7d$

$d = 6$

(x) $\ell = 28$, i.e., $t_n = 28$

$\Rightarrow t_9 = 28 \Rightarrow a + 8d = 28$

$S = 144$, i.e., $S_9 = 144$

$\Rightarrow \frac{9}{2}\{t_1 + t_9\} = 144 \quad \Rightarrow \frac{9}{2}(a + 28) = 144$

$\Rightarrow a + 28 = 32 \quad \Rightarrow a = 4$

Q4. How many terms of the AP : 9, 17, 25,.... must be taken to give a sum of 636?

Sol. $a = 9, d = 8$

Let $S_n = 636$

$\Rightarrow \frac{n}{2}[2a + (n-1)d] = 636$

$\Rightarrow \frac{n}{2}\{2 \times 9 + (n-1)(8)\} = 636$

$\Rightarrow \frac{n}{2}\{18 + 8n - 8\} = 636$

$\Rightarrow \frac{n}{2}\{8n + 10\} = 636 \quad \Rightarrow n(4n + 5) = 636$

$\Rightarrow 4n^2 + 5n - 636 = 0$

$\Rightarrow n = \frac{-5 \pm \sqrt{25 + 10176}}{8} = \frac{-5 \pm \sqrt{10201}}{8}$

$= \frac{-5 \pm 101}{8} = -\frac{106}{8}$ or $\frac{96}{8} = -\frac{53}{4}$ or 12

We reject $n = -\frac{53}{4} \Rightarrow n = 12.$

Hence, 12 terms makes the sum.

Q5. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Sol. $a = 5$, last term $t_n = 45$ and $S_n = 400$

$S_n = 400 \Rightarrow \frac{n}{2}\{t_1 + t_n\} = 400$

$\Rightarrow \frac{n}{2}\{5 + 45\} = 400 \Rightarrow \frac{n}{2} \times 50 = 400$

$\Rightarrow n = 16$

Now, $t_n = 45 \quad \Rightarrow t_{16} = 45$

$\Rightarrow a + 15d = 45 \quad \Rightarrow 5 + 15d = 45$

$\Rightarrow 15d = 40 \quad \Rightarrow d = 8/3$

Q6. The first and the last term of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Sol. Given that,

$a = 17$

$\ell = 350$

$d = 9$

Let there be n terms in the A.P.

$\ell = a + (n - 1)\, d$

$350 = 17 + (n - 1)9$

$333 = (n - 1)9$

$(n - 1) = 37$

$n = 38$

$$S_n = \frac{n}{2}(a + \ell)$$

$$\Rightarrow \quad S_n = \frac{38}{2}(17 + 350) = 19(367) = 6973$$

Thus, this A.P. contains 38 terms and the sum of the terms of this A.P. is 6973.

Q7. Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.

Sol. $d = 7$

$a_{22} = 149$

$S_{22} = ?$

$a_{22} = a + (22 - 1)d$

$149 = a + 21 \times 7$

$149 = a + 147$

$a = 2$

$S_n = \frac{n}{2}(a + a_n) \quad = \frac{22}{2}(2 + 149) = 11(151) = 1661$

Q8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Sol. $t_2 = 14, t_3 = 18$

$d = t_3 - t_2 = 18 - 14 = 4$, i.e., $d = 4$

Now $\quad t_2 = 14 \quad \Rightarrow a + d = 14$

$\Rightarrow \; a + 4 = 14 \quad \Rightarrow a = 10$

$S_{51} = \frac{51}{2}\{2a + 50d\} = \frac{51}{2}\{2 \times 10 + 50 \times 4\}$

$= \frac{51}{2} \times 220 = 51 \times 110 = 5610$

Q9. If the sum of 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of n terms.

Sol. $S_7 = 49$

$\Rightarrow \; \frac{7}{2}\{2a + 6d\} = 49 \quad \Rightarrow a + 3\,d = 7 \;...(1)$

$S_{17} = 289$

$\Rightarrow \; \frac{17}{2}\{2a + 16d\} = 289 \Rightarrow a + 8\,d = 17 \;...(2)$

Subtracting (1) from (2), we get

$5d = 17 - 7 = 10$

$\Rightarrow \; d = 2$

From (1),

$a + 3 \times 2 = 7$

$\Rightarrow$ $a = 1$

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$= \frac{n}{2}\{2 \times 1 + (n-1) \times 2\}$$

$$= \frac{n}{2}\{2n\} = n^2$$

Hence, $S_n = n^2$

Q10. Show that $a_1, a_2, \ldots a_n, \ldots$ form an AP where a_n is defined as below :

(i) $a_n = 3 + 4n$ (ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

Sol. (i) $a_n = 3 + 4\,n$

Putting n = 1, 2, 3, 4, ... in (1), we get

$a_1 = 3 + 4 = 7$, $a_2 = 3 + 8 = 11$,

$a_3 = 3 + 12 = 15$, $a_4 = 3 + 16 = 19$, ...

Thus, the sequence (list of numbers) is

7, 11, 15, 19, ...

Here, $a_2 - a_1 = 11 - 7 = 4$

$a_3 - a_2 = 15 - 11 = 4$,

$a_4 - a_3 = 19 - 15 = 4$

Therefore, the sequence forms an AP in which

a = 7 and d = 4.

$$S_{15} = \frac{15}{2}\{2a + 14d\} = \frac{15}{2}\{2 \times 7 + 14 \times 4\}$$

$$= \frac{15}{2} \times 70 = 15 \times 35 = 525$$

(ii) $a_n = 9 - 5n$

$a_1 = 9 - 5 \times 1 = 9 - 5 = 4$

$a_2 = 9 - 5 \times 2 = 9 - 10 = -1$

$a_3 = 9 - 5 \times 3 = 9 - 15 = -6$

$a_4 = 9 - 5 \times 4 = 9 - 20 = -11$

It can be observed that

$a_2 - a_1 = -1 - 4 = -5$

$a_3 - a_2 = -6 - (-1) = -5$

$a_4 - a_3 = -11 - (-6) = -5$

Therefore, this is an A.P. with common difference as –5 and first term as 4.

$$S_{15} = \frac{15}{2}[2a + (n-1)d]$$

$$= \frac{15}{2}[8 + 14(-5)]$$

$$= \frac{15}{2}(8 - 70)$$

$$= \frac{15}{2}(-62) = 15(-31)$$

$$= -465$$

Q11. If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.

Sol. $S_n = 4n - n^2$

Putting n = 1, we get $S_1 = 4 - 1 = 3$

i.e., $t_1 = 3$

$S_2 = 4(2) - (2)^2 = 8 - 4 = 4$, i.e., $S_2 = 4$

$\Rightarrow\ t_1 + t_2 = 4 \Rightarrow 3 + t_2 = 4 \Rightarrow t_2 = 1$

$t_2 - t_1 = 1 - 3 = -2 \Rightarrow d = -2$

Then $t_3 = t_2 + d = 1 - 2 = -1$, i.e., $t_3 = -1$

$t_{10} = a + 9d = 3 + 9(-2)$ $(\because t_1 = a)$

$\Rightarrow\ t_{10} = -15$

$t_n = a + (n - 1)d = 3 + (n - 1) \times (-2)$

i.e., $t_n = 5 - 2n$

Q12. Find the sum of the first 40 positive integers divisible by 6.

Sol. The positive integers that are divisible by 6 are 6, 12, 18, 24 …

It can be observed that these are making an A.P. whose first term is 6 and common difference is 6.

a = 6

d = 6

$S_{40} = ?$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{40} = \frac{40}{2}[2(6) + (40 - 1)6]$$

$= 20[12 + (39)(6)]$

$= 20(12 + 234)$

$= 20 \times 246$

$= 4920$

Q13. Find the sum of the first 15 multiples of 8.

Sol. The multiples of 8 are 8, 16, 24, 32…

These are in an A.P., having first term as 8 and common difference as 8.

Therefore, a = 8

d = 8

S_{15} =?

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{15}{2}[2(8) + (15-1)8]$

$= \frac{15}{2}[16 + 14(8)]$

$= \frac{15}{2}(16 + 112)$

$= \frac{15(128)}{2} = 15 \times 64$

= 960

Q14. Find the sum of the odd numbers between 0 and 50.

Sol. 1, 3, 5, 7 ..., 49

a = 1, d = 2

$\ell = t_n = 49$

$\Rightarrow$ a + (n – 1) d = 49

$\Rightarrow$ 1 + (n – 1) (2) = 49

$\Rightarrow$ 1 + 2n – 2 = 49

$\Rightarrow$ 2n = 50 or n = 25

The sum = $\frac{25}{2}\{a+\ell\} = \frac{25}{2}\{1+49)$

$= \frac{25}{2} \times 50 = 625$

Q15. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows : Rs. 200 for the first day, Rs. 250 for the second day, Rs. 300 for the third day, etc., the penalty for each succeeding day being Rs. 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Sol. It can be observed that these penalties are in an A.P. having first term as 200 and common difference as 50.

$a = 200$

$d = 50$

Penalty that has to be paid if he has delayed the work by 30 days = S_{30}

$S_{30} = \frac{30}{2}[2(200) + (30-1)50]$

$= 15\ [400 + 1450]$

$= 15\ (1850)$

$= 27750$

Q16. A sum of Rs. 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs. 20 less than its preceding prize, find the value of each of the prizes.

Sol. Let the Ist prize be of Rs. a.

Then the next prize will be of Rs. $(a - 20)$

Then the next prize will be of Rs. $\{(a - 20) - 20\}$,

i.e., Rs. $(a - 40)$

Thus, the seven prizes are of Rs. a, Rs. (a – 20), Rs. (a – 40), ... (an AP)

Then a + (a – 20) + (a – 40) + ... to 7 terms = 700

$\Rightarrow \frac{7}{2}\{2a + 6 \times (-20)\} = 700 \qquad (\because d = -20)$

$\Rightarrow \frac{7}{2} \times (2a - 120) = 700 \Rightarrow a - 60 = 100$

$\Rightarrow a = 160$

Thus, the 7 prizes are of Rs. 160, Rs. 140, Rs. 120, Rs. 100, Rs. 80, Rs. 60, Rs. 40.

Q17. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students?

Sol. It can be observed that the number of trees planted by the students is in an AP.

1, 2, 3, 4, 5 12

First term, a = 1

Common difference, d = 2 – 1 = 1

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{12} = \frac{12}{2}[2(1) + (12-1)(1)]$

$= 6\ (2 + 11)$

$= 6\ (13)$

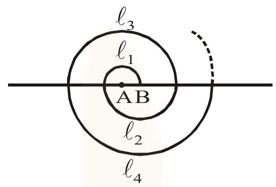
$= 78$

Therefore, number of trees planted by 1 section of the classes = 78

Number of trees planted by 3 sections of the classes = 3 × 78 = 234

Therefore, 234 trees will be planted by the students.

Q18. A spiral is made up of successive semi-circles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm,... as shown in fig. What is the total length of such a spiral made up of thirteen consecutive semi-circles? (Take $\pi = 22/7$)



Sol. From the figure,

$\ell_1 = \pi \times \frac{1}{2}$, $\ell_2 = \pi \times 1$, $\ell_3 = \pi \times \frac{3}{2}$, $\ell_4 = \pi \times 2$, and so. i.e., $\ell_1 = \frac{1}{2}\pi$, $\ell_2 = \pi$, $\ell_3 = \frac{3}{2}\pi$, $\ell_4 = 2\pi$,.....

Thus, $\ell_1, \ell_2, \ell_3, \ell_4$,........ form an AP.

$\because \quad \ell_2 - \ell_1 = \ell_3 - \ell_2 = \ell_4 - \ell_3 = ... = \frac{1}{2}\pi$

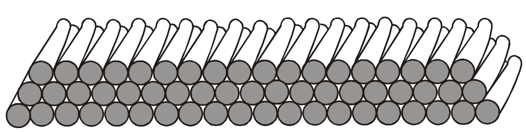
Thus, $a = \frac{\pi}{2}$, $d = \frac{\pi}{2}$

Length of the spiral $= \ell_1 + \ell_2 + ... + \ell_{13}$

$$= \frac{13}{2}\{2a + 12d\} = \frac{13}{2}\left\{2 \times \frac{\pi}{2} + 12 \times \frac{\pi}{2}\right\}$$

$$= \frac{91\pi}{2}\text{cm} = \frac{91}{2} \times \frac{22}{7}\text{cm} = 143 \text{ cm}$$

Q19. 200 logs are stacked in the following manner : 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see figure). In how many rows are the 200 logs placed and how many logs are in the top row?



Sol. It can be observed that the numbers of logs in rows are in an A.P.

20, 19, 18…

For this A.P.,

$a = 20$

$d = a_2 - a_1 = 19 - 20 = -1$

Let a total of 200 logs be placed in n rows.

$S_n = 200$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$200 = \frac{n}{2}[2(20) + (n-1)(-1)]$

$400 = n(40 - n + 1)$

$400 = n(41 - n)$

$400 = 41n - n^2$

$n^2 - 41n + 400 = 0$

$n^2 - 16n - 25n + 400 = 0$

$n(n-16) - 25(n-16) = 0$

$(n-16)(n-25) = 0$

Either $(n - 16) = 0$ or $n - 25 = 0$

$n = 16$ or $n = 25$

$a_n = a + (n-1)d$

$a_{16} = 20 + (16-1)(-1)$

$a_{16} = 20 - 15$

$a_{16} = 5$

Similarly,

$a_{25} = 20 + (25-1)(-1)$

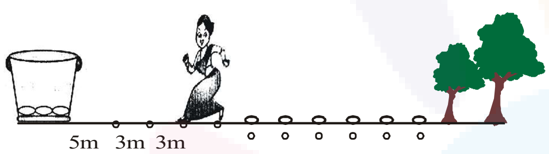
$a_{25} = 20 - 24$

$= -4$

Clearly, the number of logs in 16th row is 5. However, the number of logs in 25th row is negative, which is not possible.

Therefore, 200 logs can be placed in 16 rows and the number of logs in the 16th row is 5.

Q20. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see fig.). A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?



Sol. Distance run to pick up the Ist potato

$= 2 \times 5 = 10$ m

Distance run to pick up the IInd potato

$= 2 \times (5 + 3)$ m $= 16$ m

Distance run to pick up the IIIrd potato

$= 2 \times \{5 + 3 + 3\}$ m $= 22$ m

Thus, the sequence become 10, 16, 22, ... to 10 terms. It forms an A.P.

Here, a = 10, d = 6 and n = 10

$$\text{Sum} = S_{10} = \frac{10}{2}\{2a + 9d\} = 5 \times \{2 \times 10 + 9 \times 6)$$

$= 5 \times 74$ m $= 370$ m

Hence, the total distance run by a competitor

= 370 m.

Questions and Solutions | Exercise 5.4 - NCERT Books

Q1. Which term of the AP: is its first negative term? [Hint : Find n for an < 0]

Sol. Given AP is

121, 117, 113, ...,

Here a = 121 and d = -4

let suppose nth term of the AP is first negative team

Then,

$a_n = a + (n - 1)d$

If nth term is negative then $a_n < 0$

$\Rightarrow 121 + (n-1)(-4) < 0$

$\Rightarrow 125 < 4n$

$\Rightarrow n > \frac{125}{4} = 31.25$

Therefore, first negative term must be 32nd term

Q2. The sum of the third and the seventh terms of an AP is 6 and their product is 8. find the sum of first sixteen terms of the AP.

Sol. It is given that sum of third and seventh terms of an AP are and their product is 8

$a_3 = a + 2d$

$a_7 = a + 6d$

Now,

$a_3 + a_7 = a + 2d + a + 6d = 6$

$\Rightarrow 2a + 8d = 6$

$\Rightarrow a + 4d = 3 \Rightarrow a = 3 - 4d \quad ...(i)$

And

$a_3 \cdot a_7 = (a+2d)\cdot(a+6d) = a^2 + 8ad + 12d^2 = 8 \quad ......(ii)$

put value from equation (i) in (ii) we will get

$\Rightarrow (3-4d)^2 + 8(3-4d)d + 12d^2 = 8$

$\Rightarrow 9 + 16d^2 - 24d + 24d - 32d^2 + 12d^2 = 8$

$\Rightarrow 4d^2 = 1$

$\Rightarrow d = \pm\frac{1}{2}$

Now,

Case (i) $d = \frac{1}{2}$

$a = 3 - 4\times\frac{1}{2} = 1$

Then,

$S_{16} = \frac{10}{2}\left\{2\times1 + (16-1)\frac{1}{2}\right\}$

$S_{16} = 76$

Case (ii) $d = -\frac{1}{2}$

$a = 3 - 4\times\left(-\frac{1}{2}\right) = 5$

Then,

$S_{16} = \frac{16}{2}\left\{2\times1 + (16-1)\left(-\frac{1}{2}\right)\right\}$

$s_{16} = 20$

Q3. A ladder has rungs \small 25 cm apart. (see Fig. 5.7). The rungs decrease uniformly in length from 45 cm at the bottom to \small 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs? [Hint: Number of rungs $=\frac{250}{25}+1$]

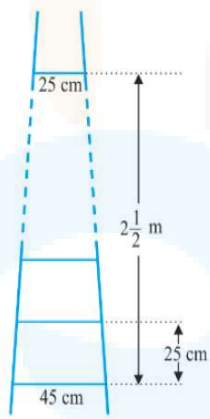


Fig. 5.7

It is given that
The total distance between the top and bottom rung

$=2\frac{1}{2}\text{m}=250\,\text{cm}$

Distance between any two rungs = 25 cm

Total number of rungs $=\frac{200}{25}+1=11$

And it is also given that bottom-most rungs is of 45 cm length and topmost is of 25 cm length. As it is given that the length of rungs decrease uniformly, it will form an AP with

$a=25, a_{11}=45$ and $n=11$

Now, we know that

$a_{11} = a + 10d$

$\Rightarrow 45 = 25 + 10d$

$\Rightarrow d = 2$

Now, total length of the wood required for the rungs is equal to

$S_{11} = \frac{11}{2}\{2 \times 25 + (11-1)2\}$

$S_{11} = \frac{11}{2}\{50 + 20\}$

$S_{11} = \frac{11}{2} \times 70$

$S_{11} = 385\,cm$

Therefore, the total length of the wood required for the rungs is equal to 385 cm

Q4. The houses of a row are numbered consecutively from 1 to 49 . Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.

[**Hint :** $S_{x-1} = S_{49} - S_x$]

Sol. It is given that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it

And 1, 2, 3, ..., 49 form an AP with **a = 1 and d = 1**

Now, we know that

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

Suppose their exist an n term such that ($n < 49$)

Now, according to given conditions

Sum of first n - 1 terms of AP = Sum of terms following the nth term

Sum of first n - 1 term of AP = Sum of whole AP - Sum of first m terms of AP

i.e.

$S_{n-1} = S_{49} - S_n$

$$\frac{n-1}{2}\{2a+((n-1)-1)d\} = \frac{49}{2}\{2a+(49-1)d\} - \frac{n}{2}\{2a+(n-1)d\}$$

$$\frac{n-1}{2}\{n\} = \frac{49}{2}\{50\} - \frac{n}{2}\{n+1\}$$

$$\frac{n^2}{2} - \frac{n}{2} = 1225 - \frac{n^2}{2} - \frac{n}{2}$$

$$n^2 = 1225$$

$$n = \pm 35$$

Given House number are not negative so we reject n = -35

Therefore, the sum of no of houses preceding the house no 35 is equal to the sum of no of houses following the house no 35

Q5. A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete. Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m. (see figh. 5.8).

Calculate the total volume of concrete required to build the terrace.

[**Hint :** Volume of concrete required to build the first step

$= \frac{1}{4} \times \frac{1}{2} \times 50 m^3$]

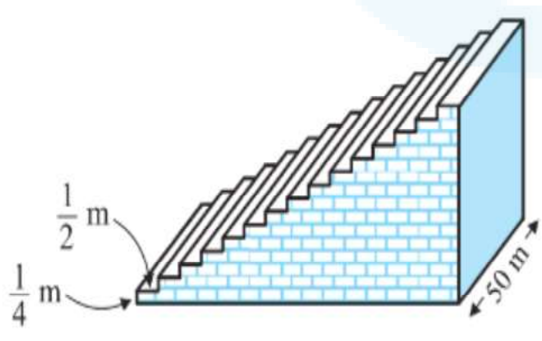


Fig. 5.8

Sol. It is given that

football ground comprises of \small 15 steps each of which is 50 m long

and Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m

Now,

The volume required to make the first step = $\frac{1}{4}\times\frac{1}{2}\times 50 = 6.25\text{m}^3$

similarly,

The volume required to make 2nd step =

$$\left(\frac{1}{4}+\frac{1}{4}\right)\times\frac{1}{2}\times 50 = \frac{1}{2}\times\frac{1}{2}\times 50 = 12.5\text{m}^3$$

And

the volume required to make 3rd step =

$$\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right)\times\frac{1}{2}\times 50 = \frac{3}{4}\times\frac{1}{2}\times 50 = 18.75\,\text{m}^3$$

And so on

We can clearly see that this is an AP with a = 6.25 and d = 6.25

Now, the total volume of concrete required to build the terrace of 15 such step is

$$S_{15} = \frac{15}{2}\{2\times 6.25 + (15-1)6.25\}$$

$$S_{15} = \frac{10}{2}\{12.5 + 87.5\}$$

$$S_{15} = \frac{10}{2}\times 100$$

$$S_{15} = 15\times 50 = 750$$

Therefore, the total volume of concrete required to build the terrace of 15 such steps is 750m^3